

Multiplication with Concrete Equipment

In school we have introduced the children to a wider range of concrete learning experiences to deepen their understanding of the concept of multiplication. We need the children to see what is happening to a number when it is multiplied to understand the effect it has on the size of the number.

In this extra multiplication booklet you will see strategies that the children are experiencing in school so that you can see the links between the concrete methods and how they support a child's understanding of formal written strategies.

We are not suggesting that you go out and buy the equipment that we show you in this booklet, but as a school we feel it is important that you see the methods we are using in the classroom so that you are able to understand and discuss your child's learning with them at home and in TLC appointments.



Introducing the word 'times' with repeated addition:

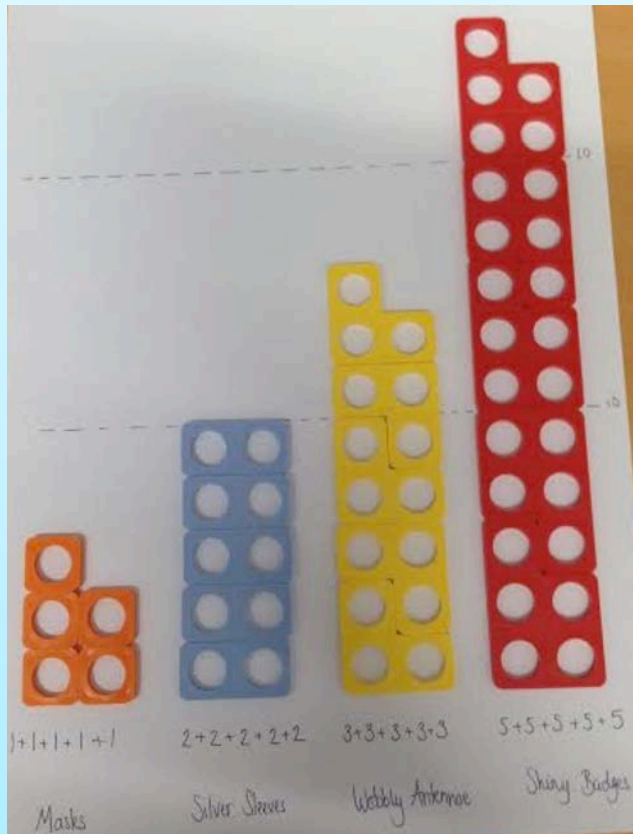
The children begin to experience multiplication formally in class in Year 2. Their first experience of this is seeing it as repeated addition. The use of real life situations can support the children's learning to see when they may need to use repeated addition.

Role Play Example:

THE SPACE PLAY

Children are told they are preparing for a class assembly and need to collect the equipment. Discuss the equipment they need: Every child needs one mask, two silver sleeves, three wobbly antennae on a head band, 5 shiny badges to wear.

The children can begin to explore using the numicon to represent the repeated addition and build the number sentences for the different equipment needed for the assembly.



The language of times can then be introduced as they build their repeated addition.

"How many times have you repeated the masks?"

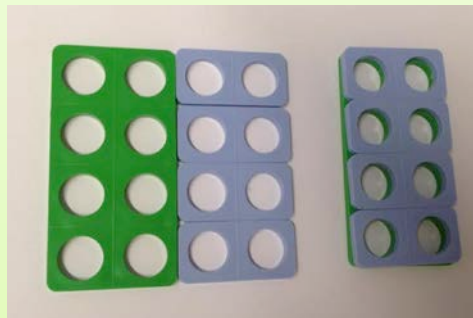
"How many times do you need to repeat the 2 shape to cover the 8 shape?"

Responses can be modeled to the children at first and then allow them time to explore with similar concepts.

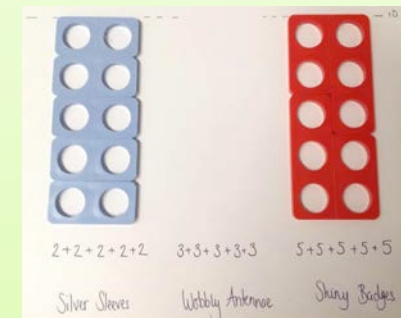
"I have repeated one plate 5 times."

"I have used the two plates 4 times to cover the 8 plate." (see picture a)

Can they spot similarities between the different plates e.g. "If I repeat the two plates 5 times, it is the same as the repeating the 5 plates 2 times. (see picture b)



Picture A



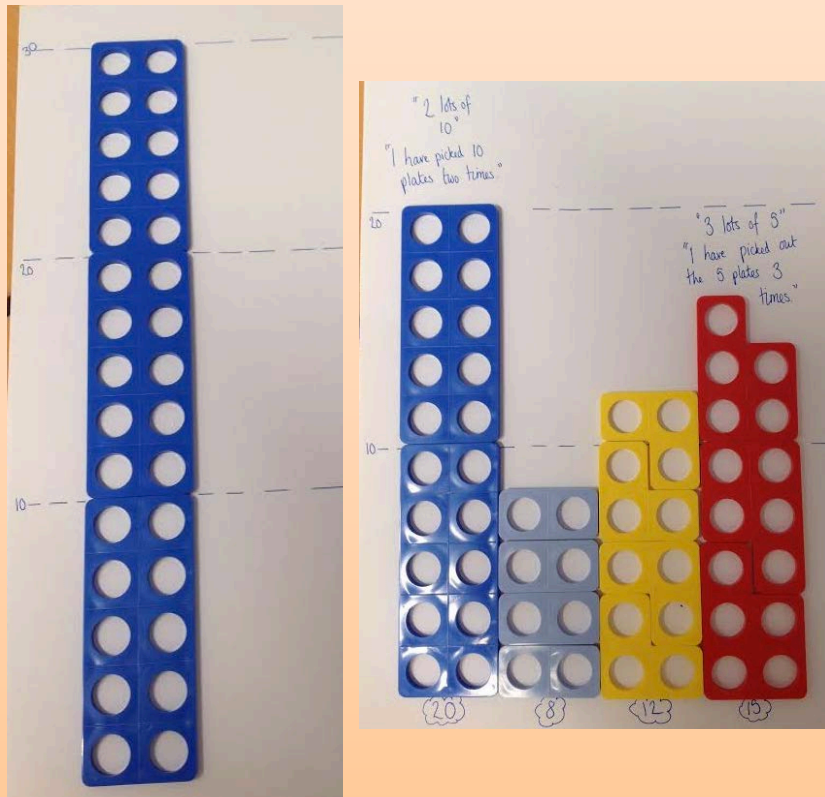
Picture B

Continuing the concept of the word 'times' using concrete equipment:

A Feely Bag Game

In a bag, that you can't see through, place a selection of 1, 2, 3, 5 and 10 plates. Ask the children to feel for the 10 plates and get them out of the bag. "How many times have you placed the ten plate in front of you?" Model responses to ensure language use, "We have placed three ten plates down, three times ten." You could then add more ten plates to discuss how many times they had picked a ten plate from the bag now.

They can then explore using other shapes in the bag, "What else is in the bag?" Get them to place the tiles together in their plates to continue to discuss how many times they have picked a particular tile.



Extension Activity:

Be specific with what you ask the children to select when they are confident to see how many times a plate has been picked out the bag. The language of times and lots of needs to be intertwined to allow them understand the concept of multiplication. Children need to be able to fluidly move between the two types of vocabulary. Continue to get the children to explain their thinking, reasoning and any observations of patterns they see.

"Can you pick me 5 lots of two?"

"Can you pick two shapes 5 times?"

Vary the shapes and the language as you go.

Ask the children to then select zero times 5. Observe what they do and discuss. Share how it is picking up the 5 tile 'no times' or 'zero times'.

Repeat with different examples and vary the language again e.g. "zero lots of 3...", "2 times zero...", "pick up a ten plate no times". Vary the use of 'no times' and 'zero' amongst asking for a different number of plates e.g. 5 lots of 3 plates etc. This will help the children to be able to work fluidly and show a deep understanding of times and lots of, whilst consolidating the understanding of zero.

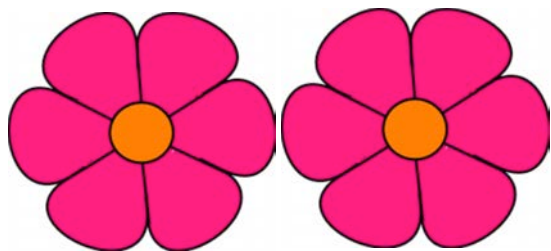


Now use objects e.g. 2 piles of 3 apples. What does this represent? See if the children can articulate it is '2 lots of 3' or 'two times three (2 lots of 3 apples).' The children again may need this modeling to them at first and discuss with a variety of language. Can they organise the equipment given e.g. 'Can you show me 3 lots of 3?' "Can you pick me 5 times two beads?"

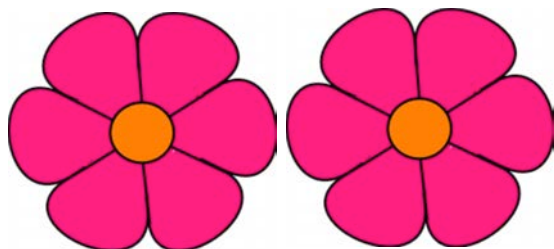
Pictorial Representation of 'times':

Images can help the children to discuss their understanding of the concept 'times'. Place them in front of the children and ask them to discuss what they see. Again encourage the terms of 'times' and 'lots of'.

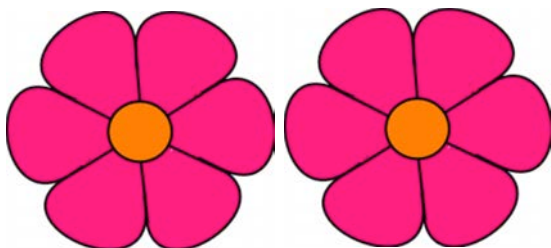
Try not to add too many variables in at this stage e.g. varying the colour or design of the images used as this could confuse the children as they will see them as different.



Two lots of three cars
Two times three cars
Two lots of three
Two times three



What can you see?
Discuss using language like
"three times two flowers",
"three times two", "three
lots of two".



Abstract Representation: Introducing the 'x' symbol.

At this point when the children are secure you can begin to discuss how saying or writing the word 'times' or 'lots of' all the time can take a long time. So this is a perfect opportunity to introduce the 'x' symbol to them. You can remake some of the concrete or abstract activities but record it with the 'x' symbol to represent the number sentences.

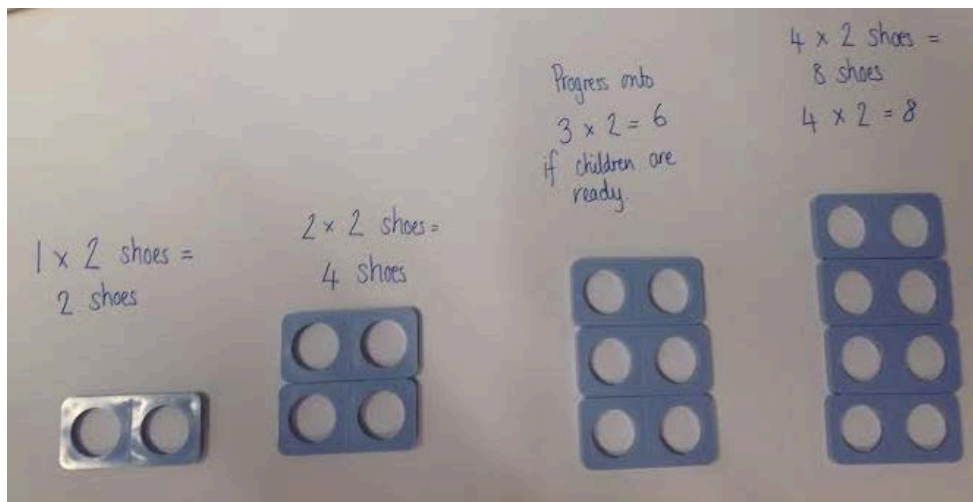
Working with the 'x' symbol:

It is a good idea to start with the 2 times table as this will not only support their consolidation of multiplication but allow them to build the skills of writing number sentences accurately.

The Noise Game:

You will need numicon plates and a container that makes a sound once a numicon plate is dropped into it. Give the children a context where there are two in a group e.g. a shoe shop, astronaut boots, gloves or socks. Tell the children a story that the people need enough equipment to wear.

The shoe shop: Tell the children the shoe shop is very busy today and customers are coming in to buy lots of shoes. Show the children a two plate and tell them this represents a pair of shoes. Then ask them to close their eyes and you drop an amount of plates in the container. The children are to listen for the sound of the plates dropping and count them as they go. Ask them to open their eyes when you have finished and use language like 'how many times did you hear a plate drop into the container?' When you have agreed on the amount of times ask them how many shoes that this represents. Model the number sentence verbally e.g. "3 times 2" or "3 lots of 2". They may want to represent this with the numicon to find the solution. Remember that the '=' means equal to so begin to use this language with your child, try to use the language "the answer is..." as children begin to think that the equals symbol just means 'answer', which can hinder their understanding of other concepts. When you go to record your number sentence make it very clear that the line is diagonal and discuss the difference between the addition sign and the multiplication sign. You could get the children to practise using their arms first of all.

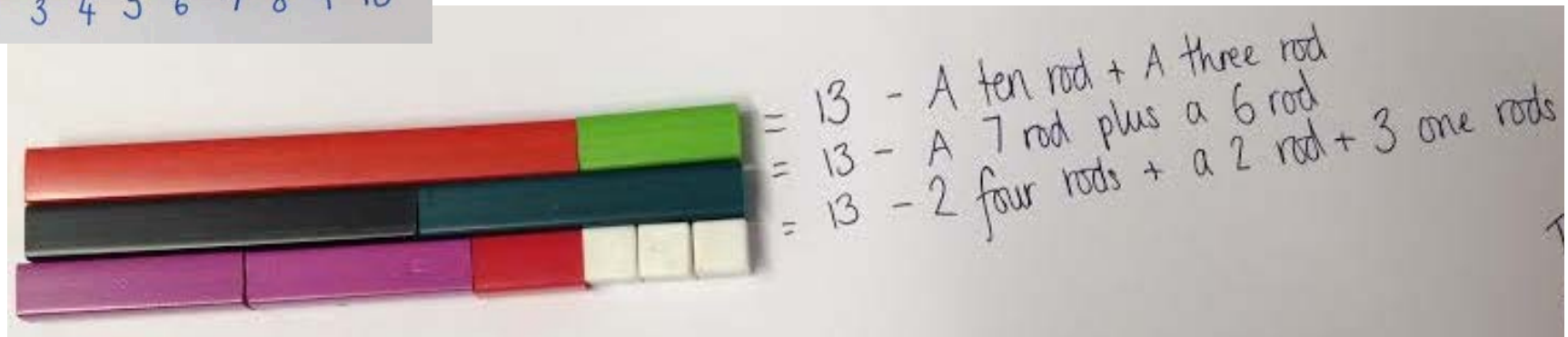
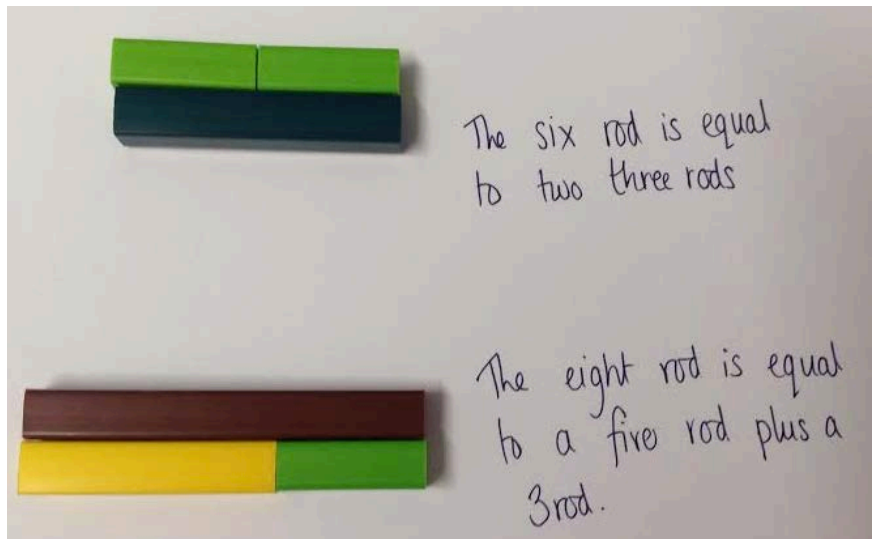
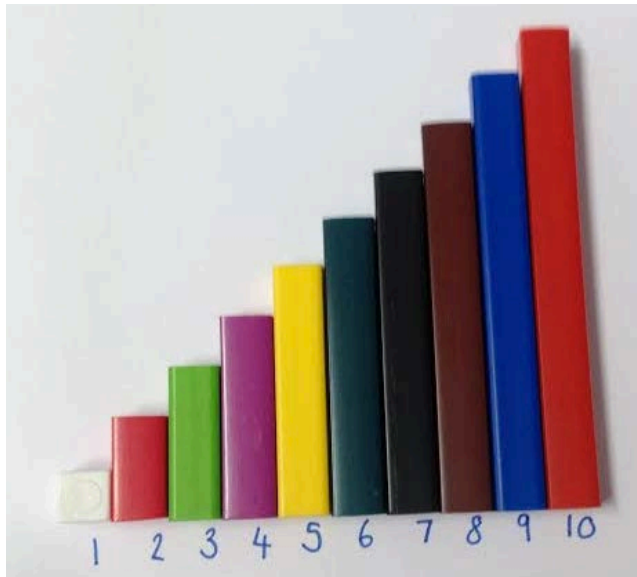


You can then repeat this with other numbers using the numicon plates. Move onto the 3 plates, then the 5's and then the 10's. It is important to discuss the patterns that the children see when recording their work. Talk about the numbers they see in the number sentences. Can they make predictions about the numbers they might see if they carry on the sequence? Do they notice any numbers that are in more than one times table? Allow them to explore and record their work in a comfortable way for them. Writing the words instead of symbols is fine as they will move to symbols when they are comfortable. You just want to ensure they are understanding the concept of multiplication.

Introducing Arrays with Concrete Equipment:

In the previous multiplication support booklet I introduced arrays as a method to work out unknown multiplication facts. This page shows how you can use concrete equipment to create these. Cuisenaire Rods are a good way to represent these and work in a similar way to Numicon however you are able to build arrays with much more ease due to the shape.

To begin with the children need to explore the rods to see patterns and relationships to them. A good start is to order them in size and they will see there are ten. This gives them their first value of the rods. This gives them an opportunity to play and seek out relationships which is key to being able to use the rods flexibly and fluidly. The rods are all proportionate in size, allowing children to see that 'the two rod is the same as two one rods' etc. Again allow your child to record in anyway they like, it could be words or number or symbols. The important part is that the Maths is accurate and they are noticing relationships. You could encourage the children to build two digit numbers and explore how many different ways that could be done.



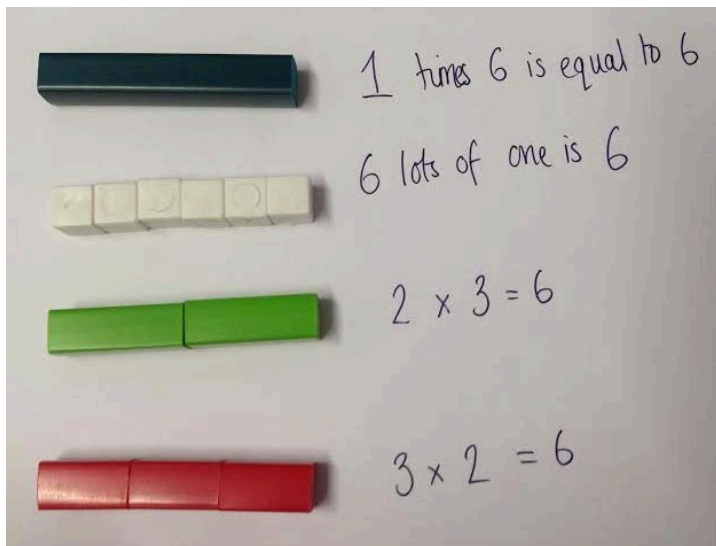
Building Arrays with Cuisenaire rods:

When children are confident and secure with manipulating the rods you can begin to use it for multiplication.

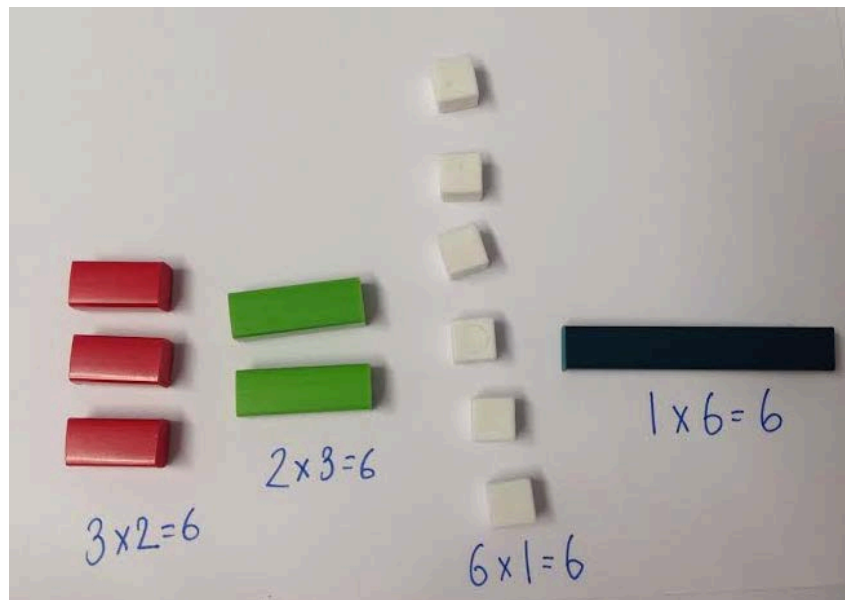
THE MOON BUGGY:

Tell the children they are going to be designing a moon buggy, which has six seats in. They need to decide how the seats will be organised. Tell them they need to be in rows that could be horizontal or vertical. The children will first need to explore the number 6 – what ways can they make it? They need to also remember that we are working with multiplication so the vocabulary ‘lots of’ and ‘times’ needs to be key. A child may pick up a 5 rod and a one rod. This needs to be discussed with them and refer them back to the vocabulary to see if this works as a concept.

Again ask the children to record what they have found out in words or symbols and have a look at what they have completed.

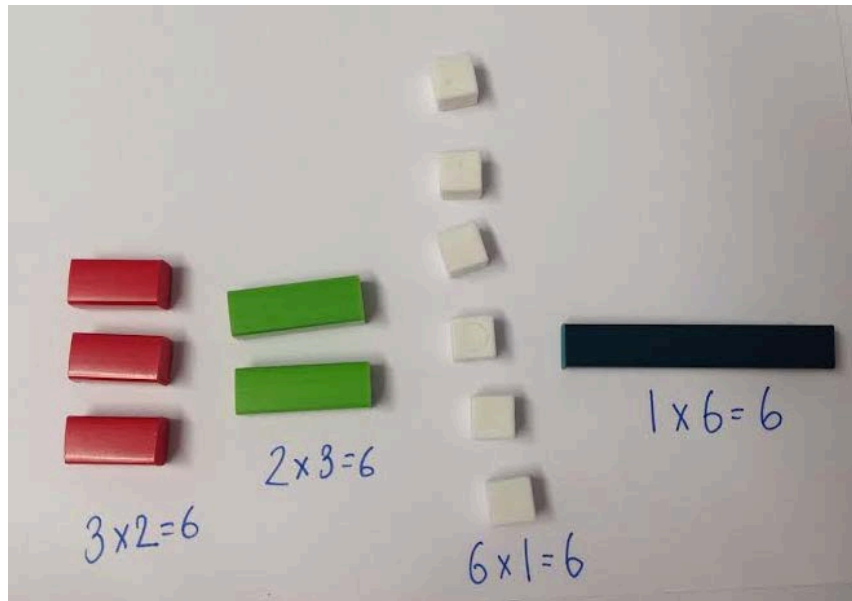


Discuss what they see and any patterns they notice. Ask them to re-order the seat horizontally, does it make a difference? Tell them to arrange the rows as if they were choosing the seating in the moon buggy. What rows of seating could we have?



We can see 3 rows of 2, 2 rows of 3, 6 rows of one or 1 row of 6.

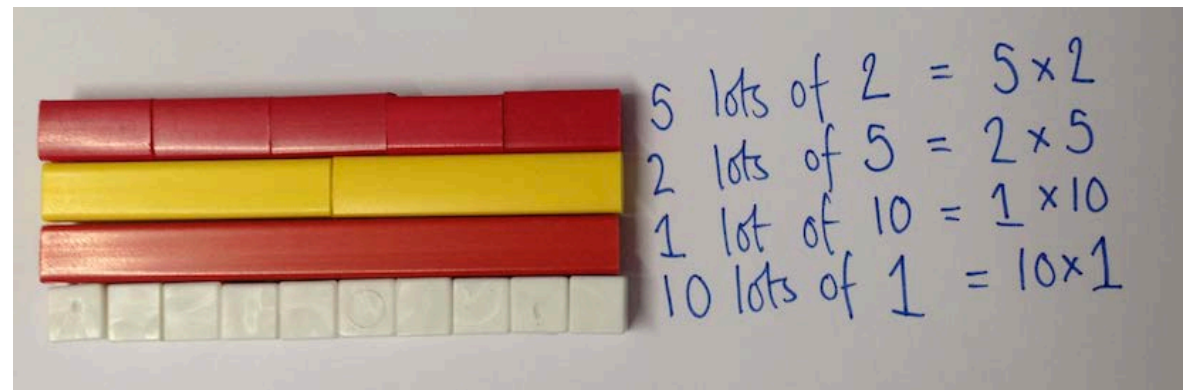
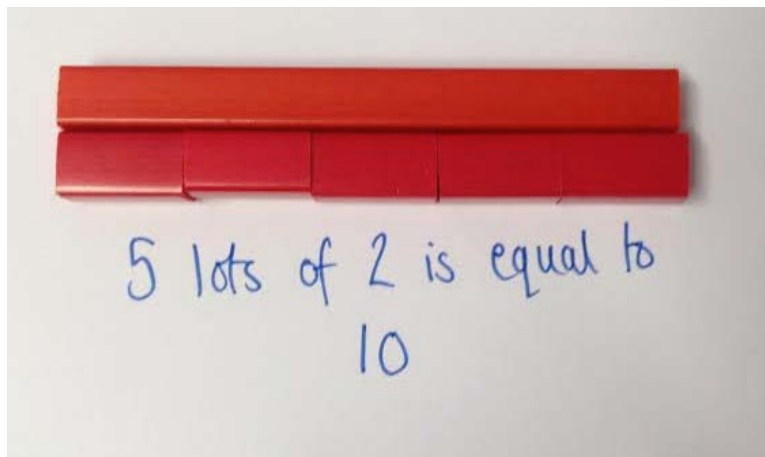
Commutative Property of Multiplication:



When discussing the patterns they see it is really important to encourage your child to notice that 2×3 is equal to 3×2 . Multiplication is commutative which means it can be done in any order. No matter which way round they multiply the two numbers they will get the same answer.

You can use the Cuisenaire rods to explore this further with different examples. E.g. 5×2 . Discuss how they would work out 5 lots of two with the rods.

Allow them time to see what other examples they can come up with to prove this. If they can realise this rule and then can test to prove it themselves the learning is so much more powerful than if the child is told and shown in an abstract form. The more they explore and find these facts out, the more their understanding will build and they will begin to trust this and rely less on equipment to check their work.

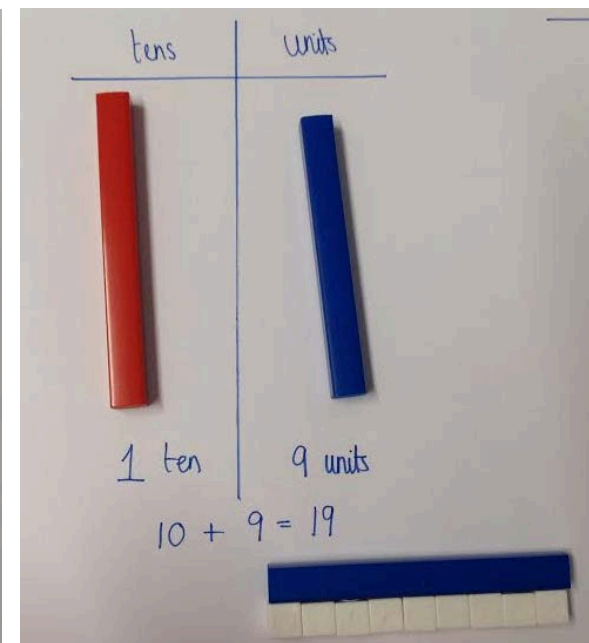
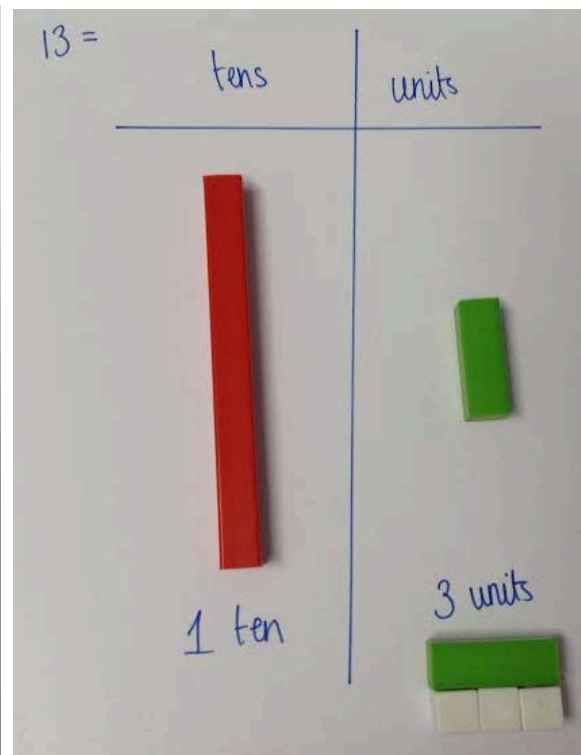
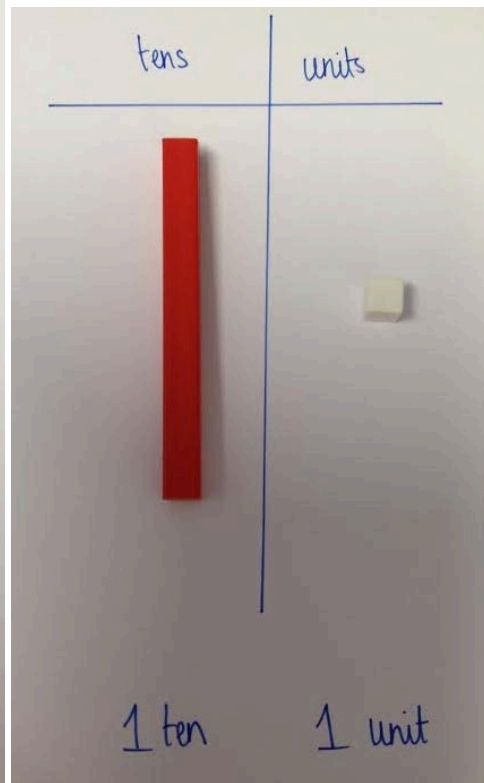


You could continue the moon buggy game with the children but just alter the number of seats in the buggy to organise the seating arrangements.

Introducing the Short Written Method:

Children will be expected to move onto formal written methods to calculate generally in Year 3 and the short written method can be learnt with the Cuisenaire Rods once more. This is a stepping stone between the concrete and the abstract so they are able to see the sense of the number at the beginning and what has happened when it has been multiplied. The children will then understand why the numbers go where they do when they are writing the formal method down, which will deepen their understanding. The key understanding children require for this is place value. Children need to be able to partition numbers quickly and fluidly into their tens and units. If a child is unable to do this and not understand the place value grids, then they will need to practise these skills further before attempting this method as they will struggle to be successful.

Partitioning numbers and using a place value grid:



Extension activities from this is to complete these without equipment and to also work with 3 digit numbers. Understanding hundreds, tens and units.

Introducing the Short Written Method:

When your child is secure with partitioning with the tens and units they are ready to work on the formal written method. This will be used when a child has been asked to multiply a 2 digit number by a one digit number at first. It is a good idea to start with teen numbers less than twenty first of all so that they are able to manipulate the rods easily.

$$14 \times 6 =$$

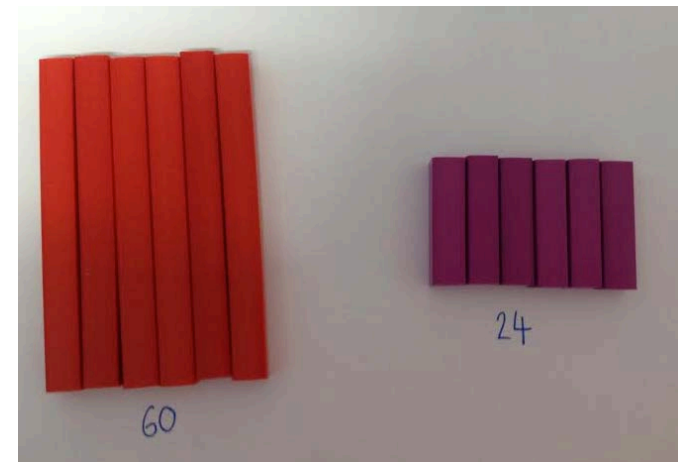
Try not to teach this as a set of instructions. Let the child explore so talk about what the questions is asking you to do. Again use that vocabulary of '14 times 6' so we are looking for '6 lots of 14'. They may suggest that they are looking for 14 lots of 6. Discuss this and let them go with it at first to consolidate the understanding that multiplication is commutative. Then tell them that you are going to be working on a new way to record their working so we are going to explore 6 lots of 14.

First the child needs to be able to make 14 using the rods and their knowledge of partitioning (one ten and 4 units). They can use the four rod or they can use the ones 4 times, either is absolutely fine.

Ask the child to explore how they can make 6 lots of this with the rods and again allow them to explore. They should hopefully make the original set they have made 5 more times to show there are six lots. If they don't keep questioning what their rods are showing them as they work and continue to encourage the language or multiplication.



Discuss how we are going to find our total. What way could we find out easily what 14×6 actually is. You are looking for your child to think about counting up all the tens and the all the fours and then finding a total from that. If they don't straight away, again allow them explore and encourage finding a 'quicker way'. Some children will attempt to add 14 six times. Discuss how this took them a while and ask them to explore again to find other ways.



Introducing the Short Written Method:

From here you can now model how we can write this as a formal method. Again try not to explain it as a set of instructions, talk about what they notice as they work. I have seen this done where it was just silently modeled to the children and they were then asked to discuss what they had seen and how it worked. They had to discuss why the numbers were being placed in a certain place without being told. Again place value is important to discuss here. The children need to understand how to partition the numbers to ensure they are placed in the correct place.

Step 1:

$$\begin{array}{r} \text{tens} \\ 14 \\ \times 6 \\ \hline \end{array}$$

Step 2:

$$\begin{array}{r} \text{tens} \\ 14 \\ \times 6 \\ \hline 4 \\ \hline 2 \end{array}$$

= 24 - 2 tens and 4 units

Step 3:

$$\begin{array}{r} \text{tens} \\ 14 \\ \times 6 \\ \hline 84 \\ \hline 2 \end{array}$$

= 60 - 6 tens

Throughout each stage discuss why numbers have been placed where they have, key questions could include:

Step 1: Why is the six under the 4? How many tens and units are in 14? Why has the symbol been written down? What do you notice about the way I have written this calculation?

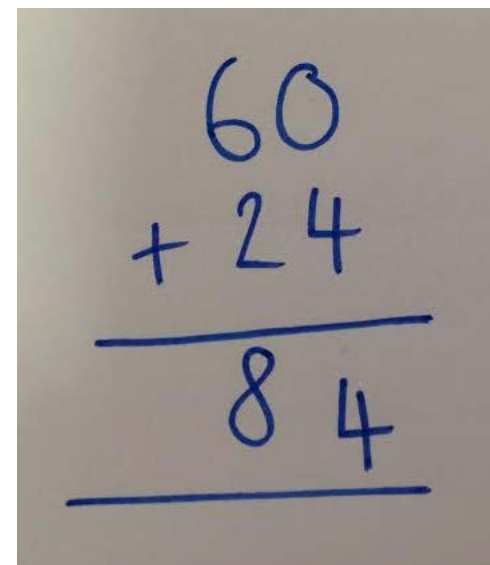
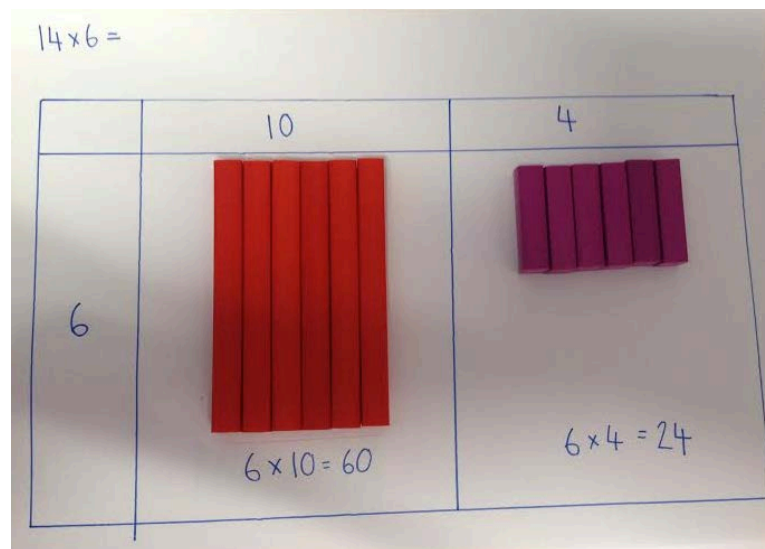
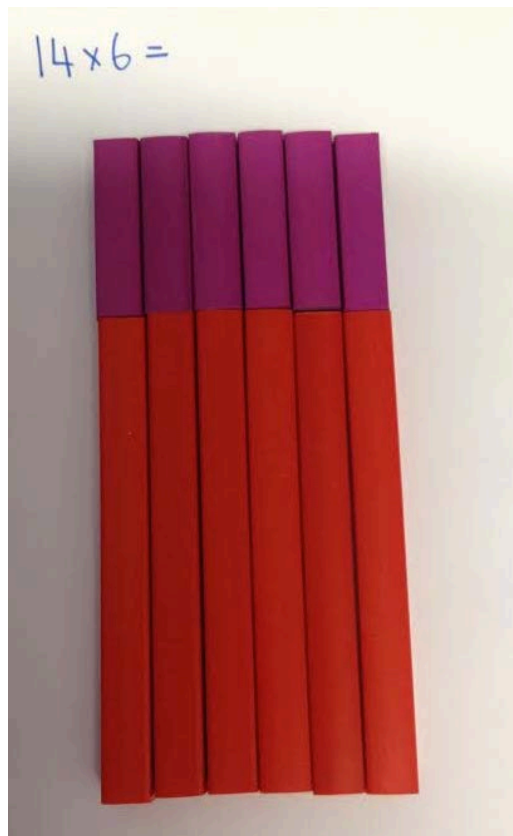
Step 2: What have we calculated here? Why do you think the number two has been written where it is? Why did we start with the units column? Why is the 4 of 24 here?

Step 3: Why is there 8 tens when we calculated 6 tens with our rods? What have we multiplied here to get our answer? Can you now explain why we started with the units column? What do I need to remember to check before I write my answer in?

You are then able to explore this further with other examples and having a go at recording this themselves. If they start with the tens column first when multiplying let them, they will soon see that it wasn't as effective as starting with the units column as they may have to edit their work. This is much more powerful than asking them to follow a set of instructions. They need to see it for themselves.

Introducing the Grid Method:

The grid method works in a very similar way to this and again the rods can be used in exactly the same way to partition the tens and units and then find the total of them. Again discuss where the numbers have been placed and why and what they notice as they work.



Place value is still key for this method and they need to build the '6 lots of 14' in exactly the same way to be able to partition it. As mentioned in the 'Multiplication Support Booklet' different children will prefer different methods but exposure to both will support their understanding of multiplication. As the children become more confident and secure the equipment will be relied upon less and they will be able to use the abstract method of just writing the numbers and symbols. Children should be secure with working abstractly as they move onto larger numbers e.g. 3 or 4 digit numbers.

